

Price Drops, Fluctuations, and Correlation in a Multi-Agent Model of Stock Markets

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Abstract

In this paper we compare market price fluctuations with the response to fundamental price drops within the Lux-Marchesi model which is able to reproduce the most important stylized facts of real market data. Major differences can be observed between the decay of spontaneous fluctuations and of changes due to external perturbations reflecting the absence of detailed balance, i.e., of the validity of the fluctuation-dissipation theorem. We found that fundamental price drops are followed by an overshoot with a rather robust characteristic time.

1 Introduction

In the recent years physicists have shown increasing interest in examining the statistical properties of real market financial data [1] and they have contributed to the extraction of the most important characteristics which are referred to as "stylized facts" [2]. Such stylized facts include fat tailed distribution and short time correlations for the logarithmic returns, volatility clustering, gain-loss asymmetry, etc.

To deepen our understanding of financial markets building models is essential. One approach is constructing purely mathematical models (e.g., ARCH and GARCH processes which are well known to and widely used by economists [3]). Another way which is more appealing for statistical physicists is that of the so-called multi-agent models. These are based on interacting agents using different strategies corresponding to real market behavior. The simplest of these models are probably the so-called "minority games" (e.g., the "El Farol bar" model [4]). Such rather abstract models are appropriate to capture main mechanisms but a detailed correspondence to economics is hard to find. An

example for more complicated models based on both economical and physical approaches is that introduced by Lux and Marchesi [5,6]. In this model a relatively large number of parameters enables to incorporate several aspects of real financial processes.

It is well known that prices on financial markets (particularly on stock markets) tend to fluctuate in a broad range. It has for long been a major goal of economists to understand the cause of these fluctuations, rises, and drops [7]. Experts seem to be puzzled by the fact that sometimes price changes can be easily traced back to well defined external effects like news about political events, announcements of dramatic economic data etc., while in many cases there is no apparent reason for the major fluctuations.

In statistical physics we distinguish between statistical fluctuations and changes due to external perturbations. If time reversal symmetry (or detailed balance) holds for the system like at thermal equilibrium, the famous fluctuation-dissipation theorem implies that spontaneous fluctuations and the response to small perturbations decay in the same way. Of course, the condition of detailed balance does not hold for financial markets. Even stationarity can be questioned and the time reversal symmetry is broken, e.g., agents want to maximize profit and tend to switch to winning strategies.

The goal of our study is to compare fluctuations and response to external effects on financial markets. In order to have a clear cut situation we investigate the the Lux-Marchesi model. The ability of the model to reproduce important stylized facts together with the relative robustness of this property when changing the parameters induced us to compare the fluctuations and response (to fundamental price changes) within this framework. We sincerely hope that our major results hold for real market data as well.

2 Simulation of the Lux-Marchesi model

Let us start with summarizing the basic features of the Lux-Marchesi single asset – multi agent model. One of the main assumptions is that there exists a fundamental price p_f of stocks (the value of the company and its prospective future growth) around which the real price fluctuates. In this model the agents are let to choose among the three following strategies: optimists (who buy whatever happens), pessimists (who sell), and fundamentalist (who sell if the market price is above the fundamental price and vice versa). Optimists and pessimists together are called chartists according to the usual terminology.

The number of all agents is $N = 500$, of which the number of fundamentalists is n_f , that of optimists n_+ , n_- for pessimists, and $n_c = n_+ + n_-$ for chartists.

The opinion index: $x = \frac{n_+ - n_-}{n_c}$ measures to what extent optimistic strategy dominates among the chartists. The aggregate excess demand of the agents for the stocks is computed as following:

$$ED = (n_+ - n_-) * t_c + n_f * \gamma * (p_f - p)$$

parameter	description
N	number of all agents
ν_1	frequency of revaluation of opinion
ν_2	frequency of transition between fund. and chart.
β	weight of demand
$T_c = N * t_c$	trade volume of chartists
$T_f = N * \gamma$	trade volume of fundamentalists
α_1	strength of herding effect
α_2	importance of price trend for chartists
α_3	importance of profit difference in transition
s	discount factor of fundamentalist profit
σ	standard deviation of μ
R	average real return of other investments
r	nominal dividend of the asset
dt	time increment in one simulation step
dt'	time increment (fast price change)
Δt	length of followed price trend

Tab. 1: The description of the parameters of the Lux-Marchesi model [6]

If the excess demand plus a small Gaussian noise term μ is positive the market-maker may increase the price by 0.01 with the probability $\pi_{\uparrow p}$; if it is negative, the price is adjusted downwards with the probability $\pi_{\downarrow p}$, where these probabilities are:

$$\pi_{\uparrow p} = \max[0, \beta(ED + \mu)]$$

$$\pi_{\downarrow p} = -\min[0, \beta(ED + \mu)]$$

The dynamics of the model is governed by the rule that agents may switch between the strategies if prospective payoffs are better using another strategy: the bigger the difference between the payoffs the higher the probability that the agent switches to the better strategy (transition probabilities are an exponential function of the profit difference). The transition probabilities between any two groups of traders are the following (where + stands for optimists, - for pessimists, and f for fundamentalists: thus e.g. π_{+-} is the transition probability of an optimist to pessimist during one time unit):

optimist – pessimist:

$$\begin{aligned}\pi_{+-} &= \nu_1 \left(\frac{n_c}{N} \exp(U_1) \right) \\ \pi_{-+} &= \nu_1 \left(\frac{n_c}{N} \exp(-U_1) \right)\end{aligned}$$

where

$$U_1 = \alpha_1 x + \alpha_2 \frac{\dot{p}}{\nu_1}$$

optimist – fundamentalist:

$$\begin{aligned}\pi_{+f} &= \nu_2 \left(\frac{n_+}{N} \exp(U_{2,1}) \right) \\ \pi_{f+} &= \nu_2 \left(\frac{n_f}{N} \exp(-U_{2,1}) \right)\end{aligned}$$

where

$$U_{2,1} = \alpha_3 \left(\left(r + \frac{\dot{p}}{\nu_2} \right) / p - R - s \left| \frac{p_f - p}{p} \right| \right)$$

pessimist – fundamentalist:

$$\begin{aligned}\pi_{-f} &= \nu_2 \left(\frac{n_-}{N} \exp(U_{2,2}) \right) \\ \pi_{f-} &= \nu_2 \left(\frac{n_f}{N} \exp(-U_{2,2}) \right)\end{aligned}$$

where

$$U_{2,2} = \alpha_3 \left(R - \left(r + \frac{\dot{p}}{\nu_2} \right) / p - s \left| \frac{p_f - p}{p} \right| \right)$$

The model uses many parameters, some of them of economic origin, some of them determining the size and reaction speed of the market (Tab. 1). The past price trend is $\dot{p}(t) = \frac{p(t) - p(t+\Delta t)}{\Delta t}$. In the simulation the price and the number of agents in each group of traders is updated after a small time interval dt , if the price change was rapid in the past couple of simulation steps the elementary time step is reduced to dt' .

In our simulation we used the parameter sets given in the original article of Lux and Marchesi (Tab. 2), if no other indication is given parameter set IV was used for the simulation.

par. set:	I	II	III	IV
N	500	500	500	500
ν_1	3	4	0.5	2
ν_2	2	1	0.5	0.6
β	6	4	2	4
T_c	10	7.5	10	5
T_f	5	5	10	5
α_1	0.6	0.9	0.75	0.8
α_2	0.2	0.25	0.25	0.2
α_3	0.5	1	0.75	1
p_f	10	10	10	10
r	0.004	0.004	0.004	0.004
R	0.0004	0.0004	0.0004	0.0004
s	0.75	0.75	0.8	0.75
σ	0.05	0.1	0.1	0.05
dt	0.01	0.01	0.01	0.01
dt'	0.002	0.002	0.002	0.002

Tab. 2: Values of the parameters in the four different parameter sets [6]

During our work we found that in the Lux-Marchesi model the autocorrelation of volatility shows exponential decay instead of a power-law time dependence in contradiction to the general view on real market data [1,8]. The power-law decay is usually attributed to some kind of scale-invariance (regarding time) in financial markets (Fig. 1) implying that real markets are in a critical state,

which we could not find in the Lux-Marchesi model even though we tried varying the parameters in a broad range. Nevertheless, the phenomenon of volatility clustering is described by the model resulting in large characteristic times of the autocorrelation function which show that the model captures important aspects of the market. Some other problems regarding the thermodynamic limit (TDL) of the model were pointed out earlier [9]. We think that these are irrelevant from our point of view since real markets (and our simulations) are far from the TDL and the time scale of the mentioned effect is bigger then that of studied fluctuations and response.

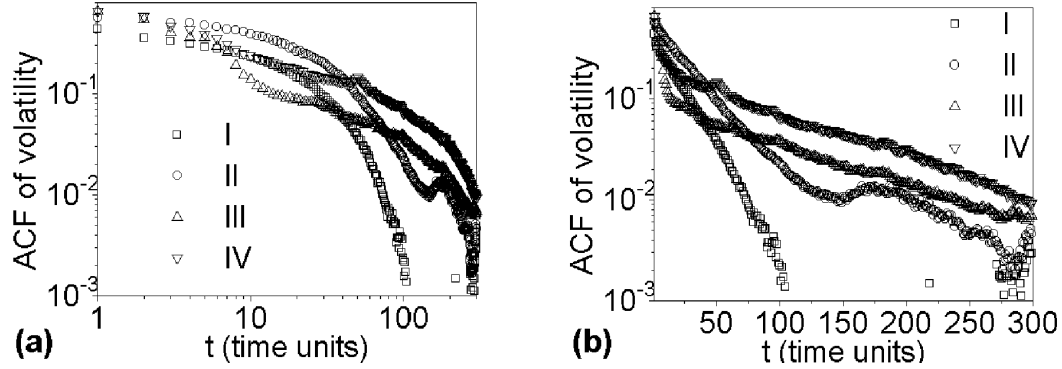


Fig. 1. Autocorrelation function of volatility for four different parameter sets (simulation length: 2 million time units) on (a) log-log and (b) lin-log scale.

3 Spontaneous price fluctuations

The dynamics of the model implies that there are continuous fluctuations around the fundamental price. We examined the decay of these fluctuations using the following simple method: when the price rose to $p = p_f + \Delta p = 10.0 + \Delta p$ we defined this as a fluctuation and observed the average decay for many runs. Hence we did not try to determine whether the price really sank after reaching $p = p_f + \Delta p$ assuming that (at least at bigger fluctuations) the probability of further rise is much smaller than that of further decline.

We observed exponential decay for the price fluctuations using all parameter sets in accordance with the well known fast decay of the correlation function. Detailed simulations were undertaken for parameter set IV (Fig. 2a). Furthermore, in case of relatively large fluctuations we observed that the opinion index and the fraction of chartists significantly differed from their average values and exponential decay was observed for both quantities. The characteristic time of the decay of the price decreased with the size of fluctuation and was in the order of magnitude of 1 time unit (up to fluctuations of 4) (Fig. 2b). The opinion index decayed with approximately the same characteristic time. On the other hand the fraction of chartists (which is closely related to volatility)

decayed much slower with characteristic times in the order of magnitude of 100 time units.

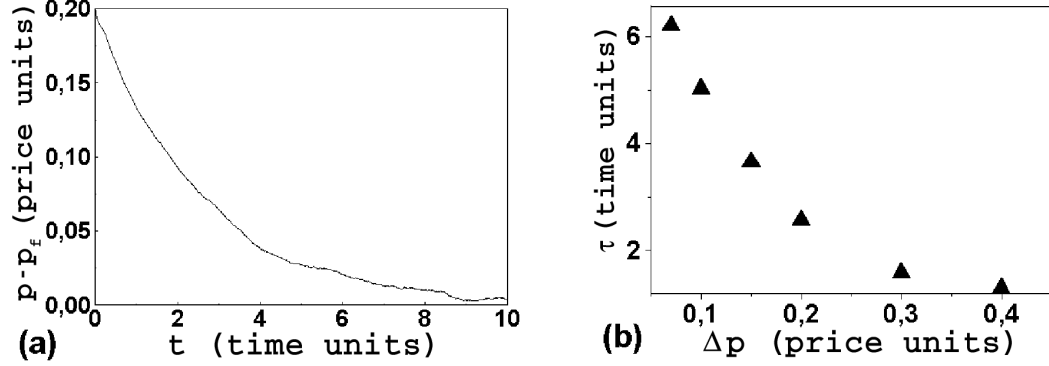


Fig. 2. (a) Average decay of 8425 price drops from 10.2 to 10.0 price units as a function of time, (b) average characteristic time of decay as a function of the size of decay (data points computed as an average of 7625 to 120529 fluctuations)

An interesting question is what causes fluctuations. In our simulations we tried to find an answer by computing the average opinion index and the average fraction of chartists that caused a fluctuation as the function of the size of the fluctuation. We found that small fluctuations are caused merely by the lack of balance inside the chartist community (Fig. 3a) while larger fluctuations are likely to occur only if the fraction of chartists to all agents rises as well (Fig. 3b) resulting in a higher market volatility [6]). In other words the fluctuation in the agents' behaviors results in high volatility, i.e. in a nervous market which occasionally leads to major deviations from the fundamental price. However, in such cases the attractive force of the fundamental price does not show up abruptly, since the re-stabilization of the equilibrium price has to be accompanied by a gradual restoration of the balance between chartists and fundamentalists.

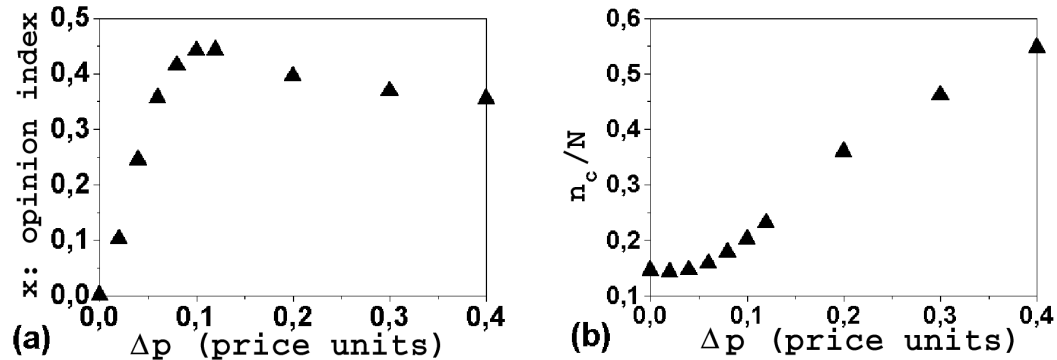


Fig. 3. (a) The average initial opinion index and (b) the average initial fraction of chartists as a function of size of the fluctuation it caused (data points computed as an average of 7625 to 120529 fluctuations)

4 Response to external effects

It is well known that financial markets are exposed to many external effects from the outside world. This means that the changes in prices are only partly due to the inherent market mechanisms (fluctuations around "equilibrium"), changes can also be caused by news from the outside world (e.g. financial reports; bankruptcy; death of important personalities; outbreak of war; terror attacks, etc.). The analysis of real market data from the point of view of external news is a highly non-trivial task. On the one hand, it is difficult to set an independent level of "importance" of news in our age of information explosion; furthermore, the effects of different news may overlap. On the other hand, the reaction of the market to the news is also hard to tell. The situation is much simpler in an artificial market like the Lux-Marchesi model where we can immediately change the fundamental value of a company or asset (which would be in reality a consequence of the external event).

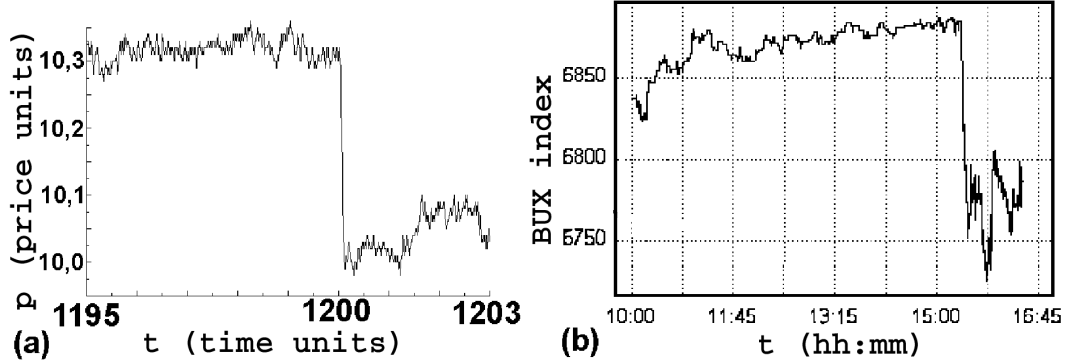


Fig. 4. (a) Price evolution in case of a fundamental price drop on simulated data (b) and the same on real market data: Budapest Stock Exchange Index (BUX) drop after the crash of flight AA587 (early rumors about terror attack) in New York at 15:17 on 12th November 2001.

In our simulation we changed the fundamental price from $p_f = 10.0 + \Delta p$ to $p_f = 10.0$ and examined the average of many runs as a function of time that has passed since the event. In the computer program we solved the averaging by raising the fundamental price by p for 100 time steps and then decreasing it to the original value, after another 100 time units we raised it again etc. This means we used a rectangular function (Θ -like function) to perturb the system and recorded the average response (Fig. 5a). What we saw in case of $p > 0$ is an abrupt drop in market price (the speed of which was only limited by the minimum time step of the model) followed by an overshoot. Economists and traders have long known that a correction exists after a very fast price change (Fig. 4). A sad example for this was the reaction of the European stock markets to the terror attack on New York on the 11th September. The prices dropped fast that day (generally losses over 10 % were recorded) but the next day there was already an upward moving trend (a correction after the

overreaction of the events).

We used two different parameter sets (II and IV) to check whether the occurrence overshoot or its shape depend on the parameters and saw that it is a rather robust effect. A detailed survey of the phenomenon was undertaken using parameter set IV. We saw that the speed of the drop in price was only limited by the model (this means a maximum of 0.05 price drop in 0.01 time steps).

Another interesting result is that (in case of a price drop) the location of the price minimum in time is independent of the price drop Δp for a wide range of Δp (for parameter set IV the price minimum is located at approximately $t = 0.42 \pm 0.02$ time units up to price drops of 10%). This means that on average one can predict when the minimum of the price occurs (if one knows the parameters of the market) irrespective of how big the fundamental price change Δp is.

Let us define the magnitude M of the overshoot as the difference of the price minimum and the new equilibrium price (which equals the fundamental price after the event). M shows linear dependence on the fundamental price change Δp in a wide range of Δp -s (up to a 10% abrupt fundamental price change, which is already huge on market scales) (Fig. 5b).

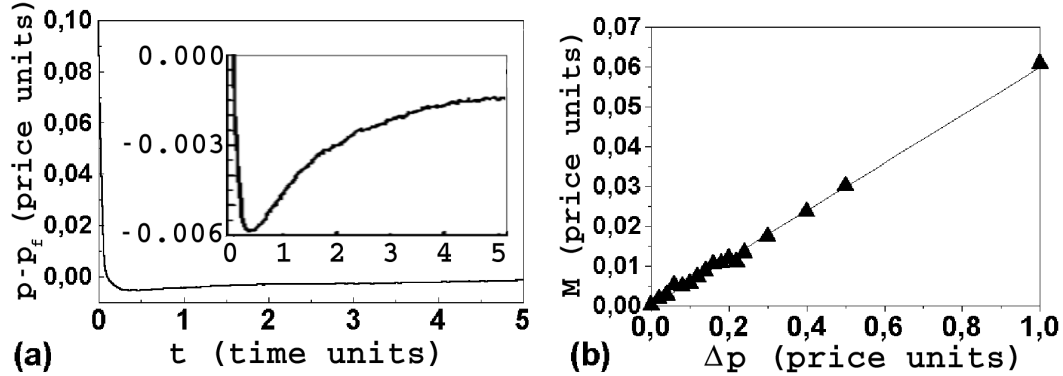


Fig. 5. (a) An average of 50000 price drops following an abrupt fundamental price change from 10.1\$ to 10.0\$; inset shows the overshoot on a different scale (b) average size of overshoot as a function of the fundamental price drop (each data point calculated as an average of 50000 price drops).

When examining the cause of overshoots within the model, the explanation is at hand: during the sudden drop of the price the proportion of pessimists rises sharply within the chartists. On the other hand, the fraction of chartists among all dealers does not change. This means that the overshoot is caused only by the movements inside the chartist "community". When the price first reaches the new equilibrium state, the fraction of pessimists is still very high which implies the further drop of price, resulting in the overshoot. Furthermore the opinion index decays to zero (after a sharp drop immediately after the event)

exponentially.

5 Discussion

The main result of the presented simulations is that in the Lux-Marchesi model of financial markets fluctuations and response to external perturbations (events) decay in a significantly different manner. Spontaneous fluctuations decay with a relatively long characteristic time while the response to external events is practically immediate and followed by an overshoot. The absence of the validity of the Onsager hypothesis for this model (and for financial markets in general) is not at all surprising since the continuous competition for profit (better payoffs) works so as to undermine the detailed balance and time reversal symmetry. The violation of the time reversal symmetry has further consequences as well, like the asymmetry of the time dependant cross correlations between different stocks [10].

The investigation of the opinion index and the fraction of chartists in case of fluctuations and responses showed that the decay mechanisms are indeed different: large spontaneous deviations from the equilibrium price occur in highly volatile markets which are accompanied by an increase of the ratio of chartists (pessimists and optimists) as compared to fundamentalists. The reaction of the market to an external change of the fundamental price is mainly governed by the shift in the ratio of the optimists and pessimists inside the group of chartists. The consequence of the latter is a well defined and rather robust overshoot in the price. The characteristic time is short and the size of the overshoot is small indicating that the market tries to adjust to the new situation effectively, however, at the same time, the phenomenon itself shows the limitations of this efficiency.

We have demonstrated that it is worth and possible to investigate the effect of external perturbations and spontaneous fluctuations separately in a model market. It would be most interesting to try to identify the origins of deviations from average behavior on real market data. Clearly, several difficulties have to be faced when trying to distinguish between the two mentioned mechanisms: The motivation of the agents is hidden, changes are not necessarily such abrupt as in the model, insider information may influence the pattern [11], effects of different news overlap, large fluctuations cover the overall behavior, etc. Nevertheless, we believe that our study gives a hint how to approach this problem.

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